Chapter 15

15.1

The reaction $\pi^0 \to \gamma + \gamma + \gamma$ is not allowed because it violates conservation of charge conjugation, while the electromagnetic interaction preserves charge conjugation

$$\mathsf{C}\left|\pi^0\right\rangle = +1\left|\pi^0\right\rangle, \quad \mathsf{C}\left|\gamma\gamma\gamma\right\rangle = (-1)^3\left|\gamma\gamma\gamma\right\rangle = -1\left|\gamma\gamma\gamma\right\rangle$$

15.2

To begin, magnetic flux is a *pseudoscalar*, as the magnetic field is a pseudovector

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A} \xrightarrow{\mathsf{P}} \mathbf{B}$$

and magnetic flux is given by the dot product between a pseudovector and a vector integrated over an even number of spatial dimensions

$$\Phi_B = \int \mathrm{d}^2x\, \mathbf{B} \cdot \hat{\mathbf{n}} \xrightarrow{\mathsf{P}} -\Phi_B$$

Angular momentum, much like the magnetic field, is a *pseudovector*, as it is the cross product of two vectors

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} \xrightarrow{\mathsf{P}} \mathbf{L}$$

Charge is a *scalar*, as it's given by the integral

$$Q = \int \mathrm{d}^3x\, \rho(x) \xrightarrow{\mathsf{P}} Q$$

for which a parity inversion is simply a change of variables. The scalar product of a vector and a pseudovector is a *pseudoscalar*. The scalar product of two vectors and of two pseudovectors, however, is a *scalar*.

15.3

The two dimensional representation of rotations in 3 dimensions is given by

$$\mathbf{R}(\hat{\mathbf{n}}, \theta) = \exp\left(-i\frac{\theta}{2}\boldsymbol{\sigma} \cdot \mathbf{n}\right) = \mathbb{1}\cos\frac{\theta}{2} - i\boldsymbol{\sigma} \cdot \mathbf{n}\sin\frac{\theta}{2}$$

where $\boldsymbol{\sigma} = \begin{pmatrix}\sigma_1 & \sigma_2 & \sigma_3\end{pmatrix}$ are the pauli matrices. With this, we have

$$\boxed{\begin{aligned} \mathbf{R}(\hat{\mathbf{x}}, \theta) &= \begin{pmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{pmatrix} \\ \mathbf{R}(\hat{\mathbf{y}}, \theta) &= \begin{pmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{pmatrix} \\ \mathbf{R}(\hat{\mathbf{z}}, \theta) &= \begin{pmatrix} \exp(-i\theta/2) & 0 \\ 0 & \exp(i\theta/2) \end{pmatrix} \end{aligned}}$$