Chapter 6

6.1

For the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{1}{2}m^2\varphi^2$$

the equations of motion are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \varphi} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) &= 0 \\ -m^2\varphi - \partial_\mu \partial^\mu \varphi &= 0 \\ \boxed{\left(\partial^2 + m^2\right)\varphi = 0} \end{aligned}$$

the conjugate momentum is

$$\boxed{\pi = \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = \dot{\varphi}}$$

and the Hamiltonian is

$$\begin{aligned} \mathcal{H} &= \pi\dot{\varphi} - \mathcal{L} \\ &= \dot{\varphi}^2 - \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\boldsymbol{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2 \\ \boxed{\mathcal{H} &= \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}(\boldsymbol{\nabla}\varphi)^2 + \frac{1}{2}m^2\varphi^2} \end{aligned}$$