

$$\begin{aligned}
 5.1) \quad G_{\mu\nu\lambda\sigma}(k) &= \sum_a E_{\mu\nu}^{(a)}(k) E_{\lambda\sigma}^{(a)}(k) \\
 &= A G_{\mu\nu} G_{\lambda\sigma} + B (G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda}) + C (G_{\mu\nu} k_\lambda k_\sigma + k_\mu k_\nu G_{\lambda\sigma}) \\
 &\quad + D (k_\mu k_\lambda G_{\nu\sigma} + k_\mu k_\sigma G_{\nu\lambda} + k_\lambda k_\nu G_{\mu\sigma} + k_\sigma k_\nu G_{\mu\lambda}) + E k_\mu k_\nu k_\lambda k_\sigma
 \end{aligned}$$

$$(i) \quad k^\mu G_{\mu\nu\lambda\sigma} = 0$$

$$(ii) \quad g^{\mu\nu} G_{\mu\nu\lambda\sigma} = 0$$

$$k^\mu G_{\mu\nu} = k^\mu (g_{\mu\nu} - \frac{1}{m^2} k_\mu k_\nu) = k_\nu - \frac{m^2}{m^2} k_\nu = 0$$

$$g^{\mu\nu} G_{\mu\nu} = g^{\mu\nu} (g_{\mu\nu} - \frac{1}{m^2} k_\mu k_\nu) = \delta_\mu^\mu - \frac{m^2}{m^2} = n-1, \quad n: \text{dimension}$$

$$k^\mu G_{\mu\nu\lambda\sigma} = C (m^2 k_\nu G_{\lambda\sigma}) + D (m^2 k_\lambda G_{\nu\sigma} + m^2 k_\sigma G_{\nu\lambda}) + E (m^2 k_\nu k_\lambda k_\sigma) = 0$$

$$C = D = E = 0$$

$$G_{\mu\nu\lambda\sigma}(k) = A G_{\mu\nu} G_{\lambda\sigma} + B (G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda})$$

$$\begin{aligned}
 g^{\mu\nu} G_{\mu\nu\lambda\sigma} &= A(n-1) G_{\lambda\sigma} + B (g^{\mu\nu} G_{\mu\lambda} G_{\nu\sigma} + g^{\mu\nu} G_{\mu\sigma} G_{\nu\lambda}) \\
 &= A(n-1) G_{\lambda\sigma} + B [g^{\mu\nu} (g_{\mu\lambda} - \frac{1}{m^2} k_\mu k_\lambda) G_{\nu\sigma} + g^{\mu\nu} (g_{\mu\sigma} - \frac{1}{m^2} k_\mu k_\sigma) G_{\nu\lambda}] \\
 &= A(n-1) G_{\lambda\sigma} + B [(\delta_\lambda^\nu - \frac{1}{m^2} k^\nu k_\lambda) G_{\nu\sigma} + (\delta_\sigma^\nu - \frac{1}{m^2} k^\nu k_\sigma) G_{\nu\lambda}] \\
 &= A(n-1) G_{\lambda\sigma} + B (G_{\lambda\sigma} - \frac{1}{m^2} k^\nu k_\lambda G_{\nu\sigma} + G_{\lambda\sigma} - \frac{1}{m^2} k^\nu k_\sigma G_{\nu\lambda})
 \end{aligned}$$

$$g^{\mu\nu} G_{\mu\nu\lambda\sigma} = [A(n-1) + 2B] G_{\lambda\sigma} = 0$$

$$n=4, \quad A = -\frac{2}{3} B$$

$$G_{\mu\nu\lambda\sigma}(k) = B [G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda} - \frac{2}{3} G_{\mu\nu} G_{\lambda\sigma}]$$

$$G_{\mu\nu}{}^{\mu\nu} = \sum_{a=1}^3 E_{\mu\nu}^{(a)} E^{\mu\nu (a)} = \delta_a^a = 5$$

$$G_{\mu\nu}{}^{\mu\nu} = g^{\mu\lambda} g^{\nu\sigma} G_{\mu\nu\lambda\sigma}$$

$$B [g^{\mu\lambda} g^{\nu\sigma} G_{\mu\lambda} G_{\nu\sigma} + g^{\mu\lambda} g^{\nu\sigma} G_{\mu\sigma} G_{\nu\lambda} - \frac{2}{3} g^{\mu\lambda} g^{\nu\sigma} G_{\mu\nu} G_{\lambda\sigma}] = 5$$

$$B [(n-1)^2 + g^{\nu\sigma} (\delta_\sigma^\lambda - \frac{1}{m^2} k^\lambda k_\sigma) G_{\nu\lambda} - \frac{2}{3} g^{\mu\lambda} (\delta_\mu^\sigma - \frac{1}{m^2} k^\sigma k_\mu) G_{\lambda\sigma}] = 5$$

$$B [(n-1)^2 + g^{\nu\sigma} G_{\nu\sigma} - \frac{2}{3} g^{\mu\lambda} G_{\mu\lambda}] = 5$$

$$B [(n-1)^2 + \frac{1}{3}(n-1)] = 5$$

$$B (9+1) = 5$$

$$B = \frac{1}{2}$$

$$G_{\mu\nu\lambda\sigma} = \sum_a E_{\mu\nu}^{(a)}(k) E_{\lambda\sigma}^{(a)}(k) = \frac{1}{2} [(G_{\mu\lambda} G_{\nu\sigma} + G_{\mu\sigma} G_{\nu\lambda}) - \frac{2}{3} G_{\mu\nu} G_{\lambda\sigma}]$$