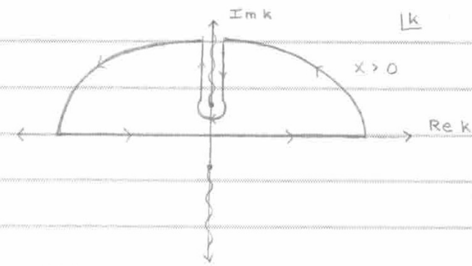
3.1) $$D(x) = -i\int \frac{d^3k}{(2\pi)^3}\,\frac{1}{2\sqrt{\vec{k}^2+m^2}}\;e^{-i\vec{k}\cdot\vec{x}}$$

$$= -\frac{i}{(2\pi)^2}\int_0^\infty dk\;k^2\int_{-1}^{1} d(\cos\vartheta)\;\frac{1}{2\sqrt{k^2+m^2}}\;e^{-ikx\cos\vartheta}$$

$$= -\frac{i}{2(2\pi)^2}\int_0^\infty dk\;\frac{k^2}{\sqrt{k^2+m^2}}\left(\frac{1}{-ikx}\right)\left(e^{-ikx}-e^{ikx}\right)$$

$$D(x) = -\frac{1}{8\pi^2 x}\int_{-\infty}^{\infty} dk\;\frac{k}{\sqrt{k^2+m^2}}\;e^{ikx}$$



$$D(x) = \frac{i}{8\pi^2 x}\,\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx}$$

$$I = \int_{-\infty}^{\infty}\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx}$$

$$= \oint\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx} - \int_{i\infty}^{im}\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx} + \int_{im}^{i\infty}\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx}$$

$$I = 2\int_{im}^{i\infty}\frac{dk}{\sqrt{k^2+m^2}}\;e^{ikx}$$

$$k = i(m+y)$$

$$I = 2\int_0^\infty dy\;\frac{1}{\sqrt{(y+m)^2-m^2}}\;e^{-(y+m)x}$$

$$u = 1+\frac{y}{m}$$

$$I = 2\int_1^\infty du\;\frac{1}{\sqrt{u^2-1}}\;e^{-mxu}$$

$$u = \cosh(t)$$

$$I = 2\int_0^\infty dt\;e^{-mx\cosh(t)}$$

$$D(x) = \frac{i}{8\pi^2 x}\,\frac{\partial I}{\partial x}$$

$$D(x) = \frac{-im}{4\pi^2 x}\int_0^\infty dt\;\cosh(t)\;e^{-mx\cosh(t)}$$

$$\sinh(t) = 0 \quad\rightarrow\quad t_0 = 0$$

$$t = t_0 + e^{i\alpha}s$$

$$\cosh(t) = \cosh(t_0) + \tfrac{1}{2}e^{2i\alpha}s^2\cosh(t_0) + \dots \approx 1+\tfrac{1}{2}s^2\;,\quad \alpha = 0$$

$$D(x) \sim \frac{-im}{4\pi^2 x}\int_0^\infty ds\;\cosh(s)\;e^{-mx(1+s^2/2)}$$

$$= \frac{-im}{4\pi^2 x}\,e^{-mx}\int_0^\infty ds\;\cosh(s)\;e^{-mxs^2/2}$$

$$\sim \frac{-im}{4\pi^2 x}\,e^{-mx}\cosh(0)\int_0^\infty ds\;e^{-mxs^2/2}$$

$$\boxed{D(x) \sim -\frac{im}{4\pi^2 x}\sqrt{\frac{2\pi}{mx}}\;e^{-mx}}$$

3.2) $$-(\partial^2+m^2)\,D(x-y) = \delta^{(2)}(x-y)$$

$$D(x-y) = \int\frac{d^2k}{(2\pi)^2}\,\frac{1}{k^2-m^2+i\epsilon}\;e^{ik(x-y)}$$

$$= \frac{-i}{2\pi}\int\frac{dk}{2\omega_k}\left[e^{-i(\omega_k t-kx)}\,\Theta(x^0) + e^{i(\omega_k t-kx)}\,\Theta(-x^0)\right]$$

$$= \frac{-i}{2\pi}\int_{-\infty}^{\infty}\frac{dk}{2\sqrt{k^2+m^2}}\;e^{-ikx}$$

$$= \frac{-i}{4\pi}\,I$$

$$D(x-y) = \frac{-i}{2\pi}\int_0^\infty dt\;e^{-mx\cosh(t)}$$

$$\sim \frac{-i}{2\pi}\,e^{-mx}\int_0^\infty ds\;e^{-mxs^2/2}$$

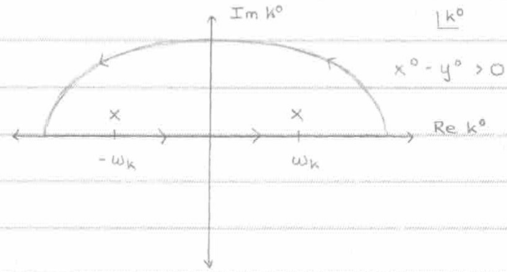
$$\boxed{D(x-y) \sim \frac{-i}{2\pi}\sqrt{\frac{2\pi}{mx}}\;e^{-mx}}$$

3.3) $$D_{adv}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 - i\,\mathrm{sgn}(k^0)\varepsilon} e^{ik(x-y)}$$

$$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{\left(k^0 - \sqrt{\omega_k^2 + i\,\mathrm{sgn}(k^0)\varepsilon}\right)\left(k^0 + \sqrt{\omega_k^2 + i\,\mathrm{sgn}(k^0)\varepsilon}\right)} e^{ik(x-y)}$$

$$D_{adv}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^0 - (\omega_k + i\varepsilon))(k^0 + (\omega_k - i\varepsilon))} e^{ik(x-y)}$$

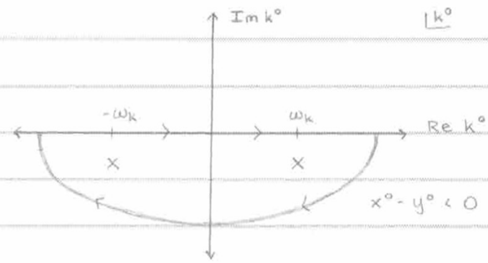
$$\omega_k^2 = \vec{k}^2 + m^2$$



$$\boxed{D_{adv}(x-y) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k^2} \left(e^{-i(\omega_k t + \vec{k}\cdot\vec{x})} - e^{i(\omega_k t - \vec{k}\cdot\vec{x})} \right) \Theta(x^0 - y^0)}$$

$$D_{ret}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 - m^2 + i\,\mathrm{sgn}(k^0)\varepsilon} e^{ik(x-y)}$$

$$D_{ret}(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^0 - (\omega_k - i\varepsilon))(k^0 + (\omega_k + i\varepsilon))} e^{ik(x-y)}$$



$$\boxed{D_{ret}(x-y) = -i \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k^2} \left(e^{-i(\omega_k t + \vec{k}\cdot\vec{x})} - e^{i(\omega_k t - \vec{k}\cdot\vec{x})} \right) \Theta(-x^0 + y^0)}$$