Problem 2.40

The relative phase induced by the magnetic field is given by $\phi = \omega T$, where $T$ is the time spent in the magnetic field and $\omega$ is given by

$$\omega = \frac{g_n|e|B}{2m_n c} \quad (1)$$

Using the de Broglie relation, $p = h/\lambda$, we can write

$$T = \frac{\ell m_n \lambda}{h} = \frac{\ell m_n \bar{\lambda}}{h} \quad (2)$$

where $\ell$ is the distance travelled in the magnetic field, and $\bar{\lambda} = \lambda/2\pi$. Combining the two previous equations, we obtain

$$\phi = \frac{g_n|e|\ell \bar{\lambda}}{2hc} B \quad (3)$$

The central maximum of the interference pattern obviously occurs when $B = 0$. To achieve the next maximum, we must have a relative phase of $\phi = 4\pi$, as is shown below

$$|\alpha\rangle = c_+ |+\rangle + c_- |\rangle \quad (4)$$

$$|\alpha'\rangle = R(\phi \hat{k}) |\alpha\rangle = c_+ e^{i\phi/2} |+\rangle + c_- e^{-i\phi/2} |\rangle \quad (5)$$

$$|\alpha'\rangle_{\phi=2\pi} = -c_+ |+\rangle - c_- |\rangle = -|\alpha\rangle \quad (6)$$

$$|\alpha'\rangle_{\phi=4\pi} = c_+ |+\rangle + c_- |\rangle = |\alpha\rangle \quad (7)$$

This yields a change in the magnetic field of

$$B = \frac{8\pi hc}{g_n|e|\lambda \ell} \quad (8)$$