Problem 2.22

(a) Since for $x > 0$ the potential is exactly that of the harmonic oscillator, and the barrier at the origin is infinite, the energy eigenfunctions in this region will consist of the H.O. energy eigenfunctions which go to zero at the origin–namely, the odd eigenfunctions. Therefore, the ground state energy will be the energy corresponding to the first odd energy eigenstate. This is realized by $n = 1$ and thus we have

$$E_G = \frac{3}{2} \hbar \omega$$

(b) To compute $\langle x^2 \rangle$, we simply need to evaluate

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} \psi_1^*(x)x^2\psi_1(x) \, dx$$

where

$$\psi_1(x) = \begin{cases} \left( \frac{2}{\sqrt{\pi}} \right)^{1/2} \left( \frac{m \omega}{\hbar} \right)^{3/4} xe^{-m \omega x^2/2h} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

Plugging this in, we obtain

$$\langle x^2 \rangle = \frac{2}{\sqrt{\pi}} \left( \frac{m \omega}{\hbar} \right)^{3/2} \int_{0}^{\infty} x^4 e^{-m \omega x^2/\hbar} \, dx$$

$$= \frac{2}{\sqrt{\pi}} \left( \frac{m \omega}{\hbar} \right)^{3/2} \left[ \frac{3\sqrt{\pi}}{8} \left( \frac{m \omega}{\hbar} \right)^{-5/2} \right]$$

$$\langle x^2 \rangle = \frac{3h}{4m \omega}$$